WEAK FORMULATIONS OF KÁRMÁN-HOWARTH-MONIN EQUATION AND THEIR APPLICATION TO TURBULENT AND CONVECTIVE FLOWS

Bérengère Dubrulle
DSM/IRAMIS/SPEC, CNRS UMR 3680, CEA, Univ. Paris-Saclay, 91190 Gif sur Yvette, France

In the classical turbulence phenomenology, valid for homogeneous flows, energy is injected at large scales, transferred downscale at a constant averaged rate ε (K41 cascade [1]) and dissipated at small scales by viscous effects [2]. This phenomenology is based on the so-called Karman-Howarth-Monin (KHM equation), derived directly from the Navier-Stokes equation. This theory was challenged by Landau, who noticed that the universal self-similarity cannot hold exactly. Indeed, in the classical energy cascade picture of turbulence, all the energy injected at large scale is gradually transferred towards smaller scale until it reaches the Kolmogorov scale $\eta = \nu^{3/4}\varepsilon^{-1/4}$ where it is dissipated by viscous processes. It is then natural to identify ε with the "viscous energy dissipation", which is subject to large fluctuations and cannot be assumed as constant. This led Kolmogorov to formulate in 1962 a refined scaling hypothesis [3], to bridge the large-scale and small-scale behavior using the quantity ε_ℓ characterizing viscous dissipation averaged over a ball of size ℓ, as

$$\varepsilon_\ell \sim \frac{(\delta u(x,\ell))^3}{\ell}, \tag{1}$$

resulting in $S_p =< \varepsilon_\ell^{p/3} > \ell^{p/3} \sim \ell^{\zeta(p)}$, showing that all the corrections to self-similarity are given by the statistics of ε_ℓ. As later argued by Kraichnan [4], if a relation like Eq. (1) holds, it should be correct only if ε_ℓ represents some well-defined local energy flux Π_ℓ, rather that space-averaged dissipation. Since Eq. (1) is not derived from NSE, there is however some vagueness in the definition of this local flux. I discuss here a derivation of the local flux from the NSE equation by a methodology similar to the one used in the derivation of the Kármán-Howarth-Monin, where the homogeneity assumption is substituted by the assumption that the velocity fields contains singularities [5]. We obtain that way a weak formulation of KHM, converging towards KHM in a suitable limit.
We then discuss application WKHM in turbulent and convective flows, regarding for example intermittency properties of the the velocity field [6], or direction of the cascades in atmospheric flows [7].

References

[1] A. N. Kolmogorov, The local structure of turbulence in incompressible viscous fluids for very large Reynolds number, *Dokl. Akad. Nauk SSSR [Sov. Phys.-Dokl.]* **30**, 913 (1941).
[2] U. Frisch, Turbulence, (Cambridge University Press, 1996).
[3] A. N. Kolmogorov, A refinement of previous hypotheses concerning the local structure of turbulence in a viscous incompressible fluid at high Reynolds number, *J. Fluid Mech.* **13**, 82 (1962).
[4] R. H. Kraichnan, Remarks on turbulence theory, *Adv. Math.* **16** 305 (1975).
[5] L. Onsager, Statistical Hydrodynamics, *Nuovo Cimento* **6 (Suppl.)**, 279 (1949).
[6] P. Debue, D. Kuzzay, D. Faranda, E-W. Saw,F. Daviaud, B. Dubrulle, Dissipation, intermittency and singularities, submitted to *Phys. Rev. E* (2018).
[7] D. Faranda, V. Lembo, M. Iyer, D. Kuzzay, S. Chibbaro, F. Daviaud, B. Dubrulle, Computation and characterization of local sub-filter-scale energy transfers in atmospheric flow submitted to *J. Atm. Sci.* (2018).

EMAIL ADDRESS FOR CORRESPONDENCE: berengere.dubrulle@cea.fr