Robust Energy Transfer Mechanism via Precession Resonance in Nonlinear Turbulent Wave Systems

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ABSTRACT

The study of exchanges of energy taking place within nonlinear wave systems has relevance for geophysical flows, oceanic waves, nuclear fusion devices and nonlinear optics. Certain efficient transfers are manifest as extreme events, localised in space and time, and can have serious consequences. The precise mechanisms by which energy is most efficiently transferred in a turbulent system remain an important open question. In this talk we present a newly discovered resonance [1] which is found to drive transfers across the spectrum of Fourier modes in a nonlinear finite-amplitude wave system. Quadratic nonlinearity of the governing PDE results in modes interacting in triads and, by considering the precessional frequencies of the 'triad phases', we show transfers are maximal when the precession resonates with the non-linear temporal frequencies. This can lead to a collective state of synchronised triads with intense cascades at intermediate nonlinearity.

Here we focus on results for the Charney-Hasegawa-Mima(CHM) equation in 2-D, given by eq. (1). This PDE has specific applications for Rossby waves in the atmosphere and drift waves in inhomogeneous plasmas.

$$(\nabla^2-\lambda)\frac{\partial\psi}{\partial t}+\beta\frac{\partial\psi}{\partial x}+\frac{\partial\psi}{\partial x}\frac{\partial\nabla^2\psi}{\partial y}-\frac{\partial\psi}{\partial y}\frac{\partial\nabla^2\psi}{\partial x}=0. \tag{1}$$

where $\quad \psi(\mathbf{x},t)=\sum_{\mathbf{k}\in\mathbb{Z}^2}A_{\mathbf{k}}(t)\mathrm{e}^{\mathbf{i}\mathbf{k}\cdot\mathbf{x}}\quad$ with Wavevector: $\mathbf{k}=(k_x,k_y)$

To facilitate our analysis we decompose the solution field into Fourier modes, where each Fourier component $A_{\mathbf{k}}(t)$ satisfies the ODE evolution equation given below in eq. (2).

$$\dot{A}_{\mathbf{k}}+i\,\omega_{\mathbf{k}}\,A_{\mathbf{k}}=\frac{1}{2}\sum_{\mathbf{k}_1,\mathbf{k}_2\in\mathbb{Z}^2}Z^{\mathbf{k}}_{\mathbf{k}_1\mathbf{k}_2}\,\delta_{(\mathbf{k}_1+\mathbf{k}_2-\mathbf{k})}\,A_{\mathbf{k}_1}\,A_{\mathbf{k}_2} \tag{2}$$

where $\omega_{\mathbf{k}}=\frac{-\beta k_x}{|\mathbf{k}|^2+\lambda}$ (linear frequencies) and $Z^{\mathbf{k}}_{\mathbf{k}_1\mathbf{k}_2}=(k_{1x}k_{2y}-k_{1y}k_{2x})\frac{|\mathbf{k}_1|^2-|\mathbf{k}_2|^2}{|\mathbf{k}|^2+\lambda}$ (interaction coefficients).

The results presented focus on heavily truncated Fourier systems where we find that the phase precession resonance mechanism is driven by the unstable manifolds of periodic orbits. This energy transfer is exposed through a rescaling of the systems initial energy that allows us to explicitly locating some of these periodic orbits and examining their trajectories with respect to the efficiency of the energy transfer. Analytically predicted precession values coincide with peak enstropy transfer as seen in Fig. 1 . Results from this small scale model are then expanded to larger systems [2] where the same quadratic non-linear interactions between Fourier modes leads to a similar correlation between phase precession frequency and Fourier mode amplitude.

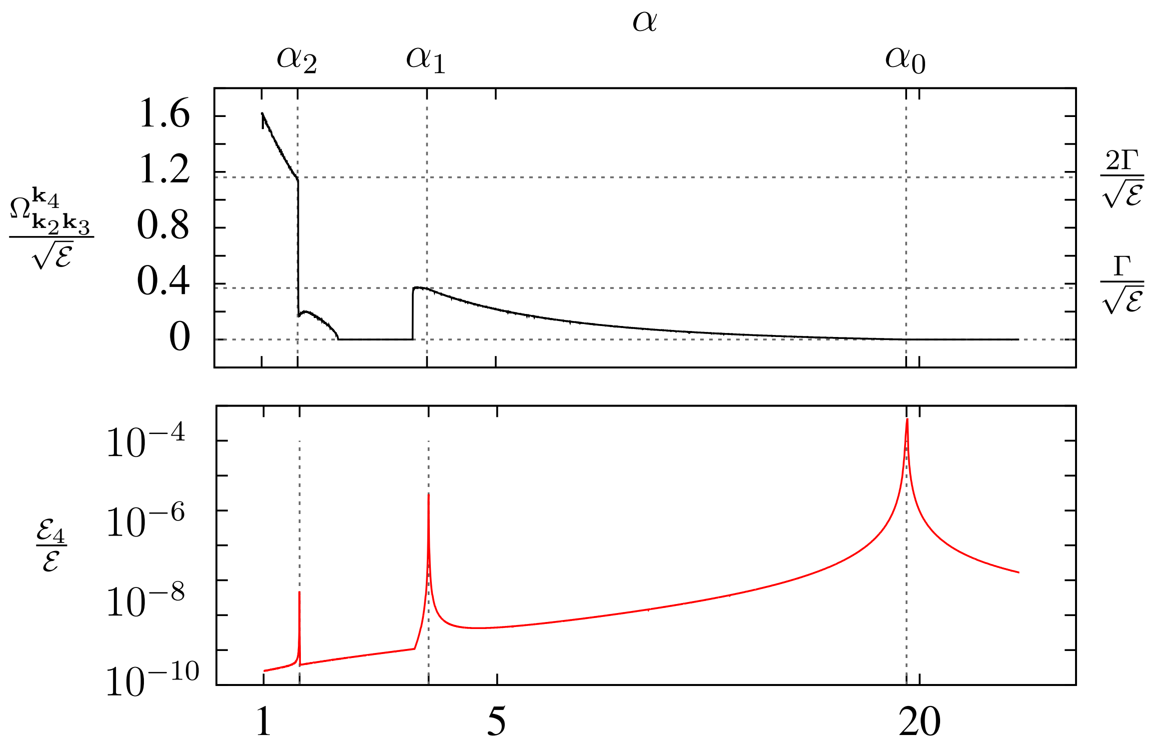

Figure 1: Numerical results for the efficiency of enstrophy $\mathcal{E}$ transfer (bottom) and value of dimensionless precession of second triad $\Omega^{\mathbf{k}_4}_{\mathbf{k}_2\mathbf{k}_3}$ (top) as a function of an initial amplitude scaling factor α. Vertical lines α_0 , α_1 and α_2 represent the analytically predicted resonances and horizontal lines $\frac{\Gamma}{\sqrt{\mathcal{E}}}$ and $\frac{2\Gamma}{\sqrt{\mathcal{E}}}$ show the harmonics of the nonlinear frequency Γ.

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[1] M. D. Bustamante, B. Quinn, & D. Lucas , Robust Energy Transfer Mechanism via Precession Resonance in Nonlinear Turbulent Wave Systems. *Physical Review Letters*, 113(8), 084502. doi:10.1103/PhysRevLett.113.084502, 2014.

[2] M. Buzzicotti, B. P. Murray, L. Biferale & M. D. Bustamante , Phase and precession evolution in the Burgers equation. *European Physical Journal E*, 39(3). doi:10.1140/epje/i2016-16034-5, 2016.