Singularity formation in thin films with generalised disjoining pressure

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A thin film coating a planar horizontal substrate may be unstable to perturbations in the film thickness due to unfavourable intermolecular interactions between the liquid and the substrate. This instability can lead to finite-time rupture of the film. The mathematical modelling of thin film rupture utilises the standard lubrication approximation, along with the disjoining pressure formalism used to account for the intermolecular interactions, to derive a single partial differential equation (PDE) for the film thickness as a function of space and time [1]. The PDE thus derived is also of theoretical interest as the physical phenomenon of rupture corresponds to singular behaviour of the governing equation [2].

Self-similar point and line rupture has been studied by numerous authors [3, 4, 5] for a particular form of the disjoining pressure with exponent $n = 3$ often used in the literature to account for van der Waals forces. A countably infinite set of similarity solutions exists, of which only one is stable, the one seen in numerical computations of the time-dependent problem [6].

Here we investigate the rupture problem for a general disjoining pressure exponent n. We compute both self-similar and fully time-dependent solutions of the governing PDE. We discover a marked change in the behaviour of the equation when n is below a critical value $n_c \approx 1.485$. At n_c, the stable branch of self-similar solutions merges with the first unstable branch, so that there are no stable solution branches below n_c. In this regime, the time-dependent computations indicate that the film no longer ruptures in a self-similar manner, instead destabilising and developing a cascading oscillatory structure as the film thins. For sufficiently small n, the film may not rupture at all. This interesting change in behaviour for different values of the disjoining pressure exponent raises questions into the theoretical properties of the PDE used to model thin film rupture, as well as the effects of different intermolecular force models, characterised by different exponents.

References

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